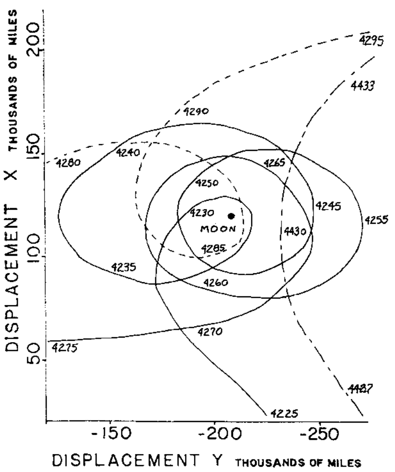

Fig. 2 Close encounter with moon (numbers refer to elapsed days).

parison of the numerical integration with analytical results for a similar two-body problem, for the same number of integration steps, showed a difference of less than 0.0025%.

The results demonstrate that under the sun's gravitational attraction, it is possible for a satellite that is placed initially at rest at a triangular libration point to escape eventually cislunar space and enter a heliocentric orbit following a close encounter with the moon. For the case studied, the motion is bounded for approximately 3700 days, during which time the satellite remains in the vicinity of the libration point. The effects of different initial conditions remain yet to be determined.

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Comment on "Choked Flow: A Generalization of the Concept and Some Experimental Data"

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In the paper[1] the authors make extensive and generous reference to a paper by myself and two co-authors.[2] However they state on p. 2177 that their equations are a general form of the one obtained by Pearson, Holliday, and Smith (PHS), but the point of view adopted and the assumptions made differ from ours. Indeed, "One of the assumptions . . . is the exact opposite of the corresponding assumption of PHS."

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Reference to p. 2179 indicates that this opposite assumption is that the authors assume J the momentum to be a minimum whereas we have assumed it to be a maximum. This latter is a simple typographical error in our paper (p. 799). In any case a simple review of the equations will reveal that we only assumed a "stationary" value to the momentum, which actually can only occur for a minimum of the momentum. There is thus no difference in the assumptions used in the two papers. I apologise for the confusion caused by our overlooking the typographical error.

Also on p. 2179, the authors appear to question our assumption that the driving stream is not isentropic and prove that if the driven stream is isentropic, the driving stream also must be. This is true in the region where both streams are flowing parallel and generalized choking occurs. We were concerned in our case with the ejector nozzle where, in the expansion process when the streams are not parallel, substantial losses due to shock waves occur. We therefore were concerned to derive the equations without assuming isentropic flow at all in the driving stream.

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Comments on Heat Induced Vibrations of Elastic Beams, Plates, and Shells

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In an article by H. Kraus,[1] which was concerned with thermally induced vibrations of thin elastic nonshallow spherical shells, the author demonstrates, as have previous authors,[2–4] that there exists the possibility of thermally induced vibrations of thin elastic beams, plates, and shells. It should be noted, however, that, since the introduction of this problem by Boley[2] in 1956, there has been no experimental evidence to support any of this analytical work. Lyons,[5] in fact, has shown that thermally induced transverse vibrations of thin elastic plates are not possible under the present conception of heat input. The present conceptions are that heat is introduced into the elastic system by prescribing on the surface of the beam, plate, or shell either an instantaneous heat flux or a temperature.

The diffusion of heat in time across the thickness of the elastic member is a necessary consideration in both of the previously considered heat inputs. Therefore, when this diffusion is eliminated in normal reduction to the one-dimensional beam problem, or the two-dimensional plate or shell problem, it is evident how these thin elastic members theoretically could exhibit vibrations because of their instantaneous surface heat inputs. This could occur since this spatial reduction imposes an infinite velocity of heat diffusion across the thickness of the elastic members.

There remains only one possible method to induce thermally vibrations in an elastic thin beam, plate, or shell. This method involves the direct instantaneous supplying of heat energy to each material element of the structural member, without depending upon thermal diffusion to transfer it.

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This conception of heat input could only be described by making use of instantaneously generated interior heat sources. Physically, this is accomplished by several means. One method is the instantaneous supplying of large amounts of electrical energy to a structure. This would allow each molecule of the structure to act as an interior heat source. A second and more important method of inducing vibrations caused by interior heat sources is the instantaneous dumping of radiation or radiation-type particles into a structure.

Consider, for example, the heat generated by gamma radiation that is applied instantaneously to an infinitely long cylindrical shell of thickness h.[6] The heat conduction in the plate is governed by the equation.[7]

$$(\partial^2 T/\partial z^2) - (\partial T/k\partial t) = -W/K \tag{1}$$

where $T(z, t)$ is the temperature in the shell, k is the thermal diffusivity of the shell material, K is the coefficient of heat conduction for the shell material, z is the space variable across the shell thickness, t is the time variable, and $W(z, t)$ defines the distribution of interior heat sources in the shell material.

If the deposition of radiation into the structural material is assumed to be constant across the thickness and instantaneous in time,

$$W = W_0 \, \delta(t) \tag{2}$$

where $\delta(t)$ is the Dirac function and W_0 is a known constant. Then the temperature in the structure will be

$$T = k \, W_0 \, H(t)/K \tag{3}$$

where $H(t)$ is the unit step function. The surfaces of the shell are assumed to be insulated perfectly.

Further, consider the equation of motion governing the motion of an infinitely long shell in its purely radial mode due to a temperature input,[8]

$$\rho h(dw^2/dt^2) + (E_p/a^2)w = (E_p/a)(1 + \nu)\alpha_t \, T_0 \tag{4}$$

where

$$T_0 = \frac{1}{h}\int_{-h/2}^{h/2} T dz \qquad \text{and} \qquad E_p = \frac{Eh}{(1 - \nu^2)} \tag{5}$$

In Eq. (4), ρ is the density of the shell material, E is the modulus of elasticity, α_t is the coefficient of linear expansion, ν is Poisson's ratio, a is the radius of shell, and w is the deflection of the shell in the radial direction, which is taken as positive outward. Substitution of (3) into (4) yields a solution,

$$w(t) = a(1 + \nu)\alpha_t(k/K)W_0(1 - \cos\omega_0 t)$$
$$\omega_0{}^2 = \frac{E}{\rho(1 - \nu^2)a^2} \tag{6}$$

The stress in the shell under this condition of heat is

$$\sigma = [E/(1 - \nu)]\alpha_t(k/K)W_0 \cos\omega_0 t \tag{7}$$

It is evident that the use of instantaneously generated interior heat sources will be the practical method for inducing vibrations in structural members. This is based on the independence of the temperature (3) on any diffusion across the thickness.

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On the Existence of a Pressure Plateau in Pure Laminar Separated Flows

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THIS note calls attention to the existence of an important discrepancy among the published works which describe the characteristics of a pure laminar, shock-induced, adiabatic, separated flow. Gadd, Holder, and Regan[1] clearly indicated that the now well-known plateau pressure exists only when transition begins prior to flow reattachment. Thus, as shown in Fig. 1a, they defined as laminar the flow for which the pressure distribution exhibited a simple reflex; whereas, they noted (as typified in Fig. 1b) that a substantial region of constant pressure upstream of the ramp signifies that the flow is becoming turbulent during reattachment. Although Chapman, Kuehn, and Larson[2] substantiated quite generally Gadd's[1] conclusion regarding the critical importance of transition in governing the nature of the shock interaction, they did not make such distinctions for the pressure distribution upstream of the ramp. In fact, their results consistently suggest that the pressure plateau is characteristic of pure laminar separation and reattachment! Because of the lack of a general appreciation of this discrepancy in the literature, considerable confusion may occur regarding the influence of Reynolds number on the upstream extent of separation for flows initially laminar. For example, Lees and Reeves[3] compared theoretical pressure distributions for ramp-induced

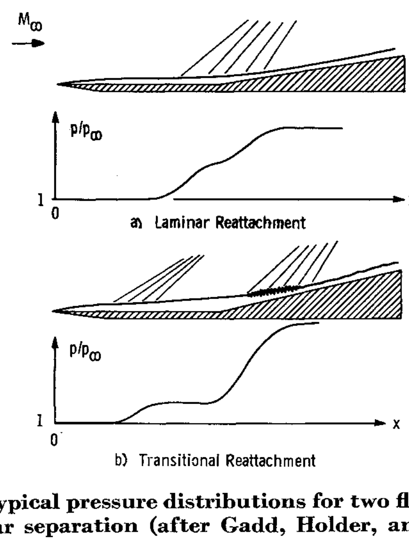

Fig. 1 Typical pressure distributions for two flow regimes of laminar separation (after Gadd, Holder, and Regan[1]).

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